The magnetic susceptibility in soft magnetic composite materials

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Abstract. The paper gives a demonstration of an analytic law which correlates the « effective » susceptibility in composite magnetic materials with the volume fraction in magnetic matter. The relation is built round two basic parameters : the first characterises the nature of the magnetic substance through an intrinsic susceptibility supposed to only be connected to the rotational magnetisation, the second characterises the heterogeneous aspect of the medium through an effective shape factor supposed to take the effects of internal demagnetising fields into account. A special attention is paid here to the latter : starting from a mixture of ellipsoidal particles, we show how the value of this average shape factor can be connected to both shapes and distributions of particles.

1. INTRODUCTION

Contrary to sintered materials in which crystals are separated by thin boundaries, composites have particles so spaced that permeability is generally lowered significantly. However they have serious advantages which come from their manufacturing processes. Powder technologies indeed are low cost and are also very attractive in terms of developments of new materials and applications as well in the microwave domain as in electrical engineering.

The laboratory currently uses magnetic powders whose particle sizes typically range from 0.1 to 150 µm. Mostly polydomains, such particles have isotropic properties quite well described by a scalar-type susceptibility. This susceptibility (noted χ_i) was introduced as an intrinsic term depending on the chemical nature of the magnetic matter used [1], while the heterogeneous aspect of the mixture was taken into account through a so-called « effective shape factor » N.

By considering the most usual case where only one type of magnetic powder is mixed with some non-magnetic binder, it was experimentally shown, both in ferrites and magnetic metals, that the magnetic behaviour of composites in weak fields, was successfully predicted by the following law [1,2]:

$$(1-N)\chi^2+\left[1+(N-C)\chi_i\right]\chi-C\chi_i=0 \qquad (1)$$

where χ denotes the effective susceptibility of the mixture, and C the volume magnetic fraction. Further investigations on various natures of magnetic powders (garnets, spinels and metals) have not only widely confirmed the intrinsic character initially attributed to χ_i, but moreover have shown it was numerically close to the rotational-type susceptibility [1].

By using an average field theory, this paper gives a theoretical development of eq.(1) first obtained phenomenologically [2]. In particular, it will be shown how the « effective shape factor » N can be connected to more tangible constants in relation with the grain shapes.

2. ABOUT EQUATION (1)

In a cartesian reference frame whose the axes are chosen to coincide with those of an ellipsoidal body of homogeneous soft magnetic matter of uniform permeability μ_i embedded in an isotropic magnetic space of uniform permeability μ, it is established that the internal field H_i resulting from application of an external field H_o has the following expression [3] :

$$H_i = [\Lambda]\, H_o \tag{2}$$

$[\Lambda]$ is a diagonal tensor whose three components δ_p are given by the next relations :

$$\delta_p = \frac{\mu}{\mu + (\mu_i - \mu) N_p} \tag{3}$$

with : $$N_p = \frac{abc}{2} \int_0^\infty \frac{ds}{(s+p^2) R_s}, \; (p = a,b,c) \quad \text{and} \quad R_s = \sqrt{(s+a^2)(s+b^2)(s+c^2)} \tag{4}$$

Factors N_p (only approachable numerically) verify the condition : $N_a + N_b + N_c = 1$.

To have an easy macroscopic description of the magnetic susceptibility in soft magnetic composite materials, actual magnetic particles are replaced by three groups of ellipsoids arranged in such a way that, in each group they have only one semi-axis (a, b or c) in coincidence with the direction of the applied field H_o. Each group of magnetic particles has the appropriate fraction (C_a, C_b or C_c) of the total magnetic load C, with of course the condition : $C = C_a + C_b + C_c$.

According to eq.(2), every group of particles has an internal field H_{ip} which is expressed as follows :

$$H_{ip} = \delta_p\, H_o \tag{5}$$

By weighting now the field components with respect to their volume fractions, an « average field » is formed whose action on the material is supposed to be similar to that of the field H_o itself :

$$H_o = \sum_p C_p\, H_{ip} + (1-C)\, H_{iv} \tag{6}$$

H_{iv} is then introduced as the field which acts on the non-magnetic fraction $(1-C)$. It will be supposed to have the same expression as the other fields, with the particular assumption : $\mu_i = 1$.

$$H_{iv} = \frac{\mu\, H_o}{\mu + (1-\mu) N} \tag{7}$$

N is a scalar parameter which will be discussed in more details further. Introducing eqs.(3) and (7) in eq.(6), leads to a formulation type well known in the Effective Medium Theories (EMT) [4] :

$$\sum_p \frac{\mu\, C_p}{\mu + (\mu_i - \mu) N_p} + \frac{\mu (1-C)}{\mu + (1-\mu) N} = 1 \tag{8}$$

With the goal of obtaining a simple and useful tool in modelling soft magnetic composite materials, we postulate first, all magnetic particles have the same chemical composition (no dispersion in μ_i), and second, the ellipsoid distribution in the three directions of space has a scalar representation simply characterised by the triplet of parameters (C, μ_i, N). Eq.(8) is then advantageously replaced by the next expression :

$$\frac{\mu\ C}{\mu+(\mu_i-\mu)N}+\frac{\mu(1-C)}{\mu+(1-\mu)N}=1 \qquad (9)$$

Note that eq.(9) can be transformed in eq.(10) which directly gives eq.(1), what ends the first part of the demonstration.

$$\frac{C(\mu_i-\mu)}{\mu+(\mu_i-\mu)N}+\frac{(1-C)(1-\mu)}{\mu+(1-\mu)N}=0 \qquad (10)$$

3. ABOUT THE EFFECTIVE SHAPE FACTOR

We remind that the substitution of eq.(8) by eq.(9) implied the following equality :

$$\sum_p \frac{C_p}{\mu+(\mu_i-\mu)N_p}=\frac{C}{\mu+(\mu_i-\mu)N} \qquad (\text{ with : } C=\sum_p C_p\ ) \qquad (11)$$

Eq.(11) assigns a physical sense to the « effective shape factor » N. Starting now from the assumption that the material investigated must have isotropic properties (no preferential orientations), conditions $(C_a=C_b=C_c=\frac{C}{3})$ are added, so that eq.(11) becomes :

$$\sum_p \frac{1}{1+\alpha N_p}=\frac{3}{1+\alpha N} \qquad (\text{ with : } \alpha=\frac{\mu_i}{\mu}-1\ ) \qquad (12)$$

where α $(0\leq\alpha\leq\mu_i-1)$ is called « factor of contrast ». Solving eq.(12) leads to :

$$N=\frac{1+2\alpha\ (N_aN_b+N_aN_c+N_bN_c)+3\alpha^2N_aN_bN_c}{3+2\alpha+\alpha^2(N_aN_b+N_aN_c+N_bN_c)} \qquad (13)$$

which clearly shows that N depends not only on the particle shapes and the nature of the magnetic substance, but also on this factor of contrast. As an example, typical cases can be examined :

3.1. Spherical particles

They are characterised by the conditions : $N_a=N_b=N_c=\frac{1}{3}$. Eq. (13) gives a solution $(N=\frac{1}{3})$ independent of α. Indeed, measurements achieved on spherical particles of Sendust (about 50 μm) have shown excellent agreements with this theoretical result [5].

3.2.Needle particles

They are characterised by the conditions : $N_a=N_b=\frac{1}{2}$, and $N_c=0$. In this case, the result becomes dependent of the factor of contrast α :

$$N=\frac{2(\alpha+2)}{\alpha^2+8\alpha+12} \qquad (14)$$

Except for the borderline case $\alpha = 0$, let us notice that N is smaller than 0.33.

To a certain extent, this result still concerns materials with spherical particles because it is known that mixtures without chain-like aggregates are very difficult to be obtained. This may explain why actual materials generally show N less than 0.33, particularly when the magnetic load is small ($C \leq 0.1$) [5]. Starting from usual values ($\mu_i = 26$ and $\mu = 9$) [2], eq.(14) gives $N = 0.25$ which is close to the value 0.27 found in experimental studies [2].

3.3. Flaky particles

They are characterised by the conditions ($N_a = N_b = 0$, and $N_c = 1$). In this case $\alpha = 2$ leads to $N = 0.14$.

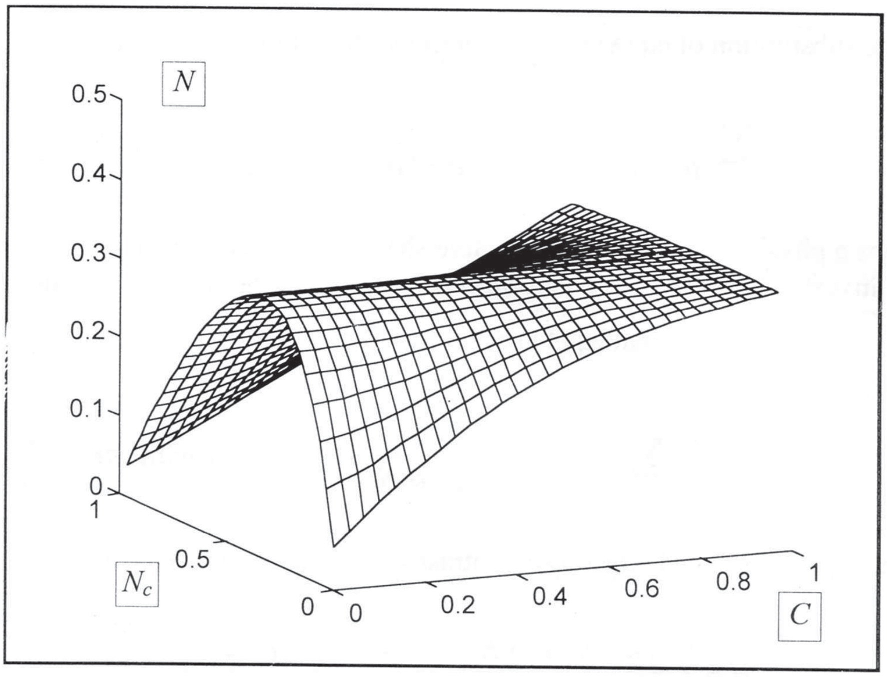

Figure 1: Representation of eq. (13) in a 3D space as a function of both the magnetic fraction C, and the particle shape N_c by considering ellipsoids of revolution around their c - axis. Let notice that needles ($N_c \cong 0$) or flakes ($N_c \cong 1$), distributed at random, give an average shape factor N which can be small in weak concentrations. Whatever the case considered, N tends to 0.33 at $C = 1$.

4. CONCLUSION

Except for non aggregated spherical particles, the second part of this work has shown that isotropic magnetic composites can be characterised by a scalar shape factor whose value is less than 0.33. It depends theoretically on both the nature and the volume fraction of the magnetic phase. Figure (1) obtained for ellipsoids of revolution summarises different results discussed previously.

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